Notes/Tricky Questions from Examlet 1:

$\forall x \in \mathbb{R}$, if $\pi = 3$, then $x < 20$.
(π is the familiar constant.) true ☑ false ☐ undefined ☐

True because of vacuous truth

1. (5 points) Suppose $\log_k x = 5$. Then $\log_k(kx^{-3}) =$

Solution: $\log_k(kx^{-3}) = \log_k k + \log_k x^{-3} = 1 + \log_k x^{-3} = 1 + -3\log_k x = 1 + -3 \cdot 5 = -14$

Remember log manipulation rules

For any real number x, $\lfloor x \rfloor < \lceil x \rceil$. true ☐ false ☑

False because x could be an integer like 2

For all positive integers n,
if $n! < -10$, then $n > 8$. true ☑ false ☐ undefined ☐

True because of vacuous truth

0! 0 ☐ 1 ☑ -1 ☐ undefined ☐

Memorize this

For all integers n, if $n^2 = 101$,
then $n > 11$. true ☑ false ☐ undefined ☐

Vacuous truth

3. (5 points) Suppose that m and p are positive integers such that $2p^2 + mp < 6$. What are the possible values for m? Briefly explain or show work.

Solution: Since $2p^2 + mp < 6$, $mp < 6 - 2p^2$. Since p is a positive integer $2p^2 \geq 2$. So $6 - 2p^2 \leq 4$. So $mp < 4$. Since m is a positive integer, this implies that m is 1, 2, or 3.

Refresh basic algebraic logic rules

$\log_3 2 \leq \log_2 3$ true ☑ false ☐

Basic log manipulation