Section 2 (Logic):

- **proposition:** statement which is either true or false
    - **2 < 15**
    - Urbana is in Illinois.
- $p \wedge q$ means p **AND** q
    - p and q is only true if **BOTH** p and q are true

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- $p \vee q$ means p **OR** q
- p or q is true when either p or q are true, or if both p and q are true (inclusive OR)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- $\neg p$ means **NOT** p
    - $\neg p$ is true when p is false

- **Conditional statement:** if p, then q
    - p is the hypothesis and q is the conclusion
    - also “p implies q” and “q follows from p”

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

- for p → q to be **FALSE**, p must be true and q must be false. If the hypothesis is false to begin with, the overall statement is true using vacuous truth. If the hypothesis is true, and the conclusion is also true, then the overall statement is also true.
- **Converse** of p → q is q → p. They are **NOT** equivalent.

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

- **Biconditional** is p ←→ q or "p implies q and conversely" means that p and q are true under exactly the same condition

p	q	$q \leftrightarrow p$
T	T	T
T	F	F
F	T	F
F	F	T

- **Contrapositive** of p → q is formed by swapping p and q and negating both of them to get $\neg q \rightarrow \neg p$
  - **EQUIVALENT** to original statement p → q

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

- If there are k variables, a truth table needs 2^k lines to cover all possible combinations of input truth values
- Two propositions p and q are **logically equivalent** if they are true for exactly the same input values, p ≡ q
- **SUPER USEFUL:** p→q is logically equivalent to $\neg p \vee q$
- **De Morgan's Laws:** $\neg(p \wedge q) = \neg p \vee \neg q$
  $\neg(p \vee q) = \neg p \wedge \neg q$
- When comparing true/false values, use ≡
- **Key equivalences**:

$$\neg(\neg p) \equiv p$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \rightarrow q) \equiv p \wedge \neg q.$$

- **Universal quantifier:** "for all", noted as $\forall$
  - $\forall x \in R$ (for all values x in R)
- **Existential quantifier:** "there exists", noted as $\exists$
  - $\exists y \in Z$ (there exists a value y in Z)
- When negating quantifiers, switch operator and move negation across:
  - $\neg(\forall x, P(x)) = \exists x, \neg P(x)$
    $\neg(\exists x, P(x)) = \forall x, \neg P(x)$