Section 3 (Proofs)

- For a claim in the form $\forall x \in A, P(x)$, choose some representative value for x. Use the fact that x is an element of A to show that P(x) is true.
- For proving some integer k as **odd**, set k=2m**+1**, where m is an integer
- For proving some integer j as **even**, set j=2n, where n is an integer
- **Order of proof:**
  - Known information from variable declarations
  - Hypothesis of the if/then statement
  - Move towards the conclusion of the if/then statement
- **Proof methods:**

	prove	disprove
universal	general argument	specific counter-example
existential	specific example	general argument

- Remember to use a different variable name in every new definition so you don't force two values that aren't necessarily equal to be the same
- At the end of the proof, write Q.E.D., (Quod erat demonstrandum)
- **Proofs by cases:**
  - If a part of the given information includes "or", you might need to test possible cases. For example, |x| > 6 means that x > 6 or x < -6, so you would need to test both cases.
  - You may need to test an unknown integer as being positive or negative, being even or odd, in different cases
- **Rephrasing claims:**
  - In order to form a direct proof, you might have to rewrite a claim so that it forms a convenient if/then statement
  - Start with this original claim (claim 10):

Claim 10 *There is no integer k such that k is odd and k^2 is even.*

Claim 11 *For every integer k, it is not the case that k is odd and k^2 is even.*

By DeMorgan's laws, this is equivalent to

Claim 12 *For every integer k, k is not odd or k^2 is not even.*

Since we're assuming we all know that even and odd are opposites, this is the same as

Claim 13 *For every integer k, k is not odd or k^2 is odd.*

And we can restate this as an implication using the fact that $\neg p \vee q$ is equivalent to $p \rightarrow q$:

Claim 14 *For every integer k, if k is odd then k^2 is odd.*

Proof by Contrapositive:

- If a claim can't be proved directly, try switching it to it's contrapositive
  - Ex. $\forall x, P(x) \rightarrow Q(x) = \forall x, \neg Q(x) \rightarrow \neg P(x)$